

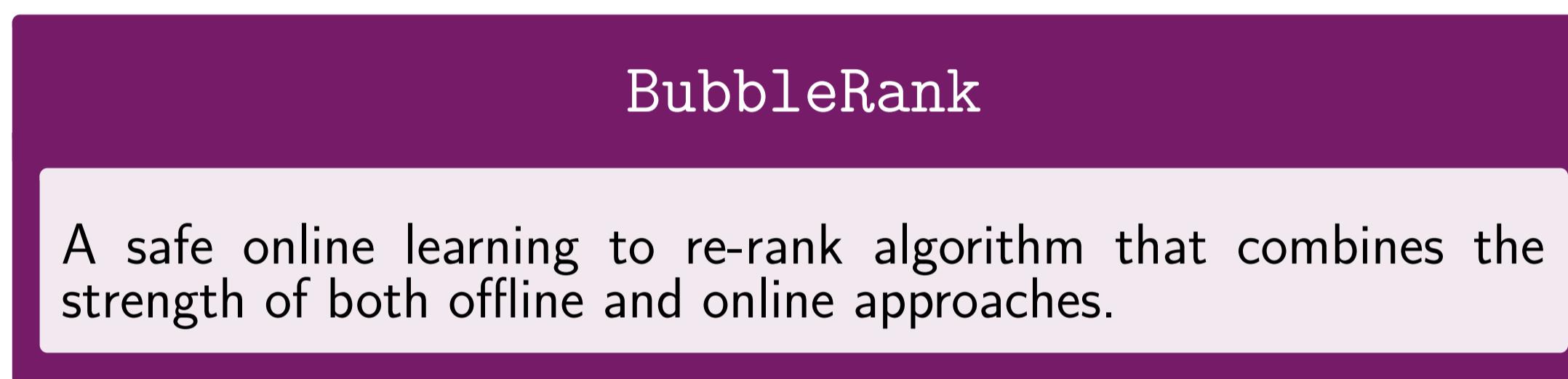
# BubbleRank: Safe Online Learning to Re-Rank via Implicit Click Feedback

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## Motivations

- ▶ Learning to rank: using machine learning to build ranking systems
- ▶ Offline approaches *lack exploration* and are limited to the provided training data
- ▶ Online approaches:
  - ▷ Balance exploration and exploitation
  - ▷ Learn from the user feedback, e.g., clicks
  - ▷ Tend to learn from scratch: *safety issue*



## Setup

- ▶ Item set  $\mathcal{D} = [L]$  and  $K \leq L$  positions
- ▶ Action set  $\mathcal{R} \in \Pi_K(\mathcal{D})$ :  $\mathcal{R}(k)$  is the item at position  $k$
- ▶ Click at any  $k \in [K]$ :  $c_t(k) = \mathbf{X}_t(\mathcal{R}_t, k)\mathbf{A}_t(\mathcal{R}_t(k))$
- ▶ Each time step  $t$ :
  - ▷ The agent chooses an action  $\mathcal{R}_t \in \Pi_K(\mathcal{D})$
  - ▷ Observe the click feedback  $c_t \in \{0, 1\}^K$
- ▶ Goal: maximize the expect number of clicks in top  $K$  positions
- ▶ Or minimize the expected cumulative regret in  $n$  steps:

$$R(n) = \sum_{t=1}^n \mathbb{E} \left[ \max_{\mathcal{R} \in \Pi_K(\mathcal{D})} r(\mathcal{R}, \alpha, \chi) - r(\mathcal{R}_t, \alpha, \chi) \right].$$

## Assumptions

For any lists  $\mathcal{R}, \mathcal{R}' \in \Pi_K(\mathcal{D})$  and positions  $k, \ell \in [K]$  such that  $k < \ell$ :

- A1.  $r(\mathcal{R}, \alpha, \chi) \leq r(\mathcal{R}^*, \alpha, \chi)$ , where  $\mathcal{R}^* = (1 \dots K)$  is the optimal ranking;
- A2.  $\{\mathcal{R}(1), \dots, \mathcal{R}(k-1)\} = \{\mathcal{R}'(1), \dots, \mathcal{R}'(k-1)\}$   
 $\implies \chi(\mathcal{R}, k) = \chi(\mathcal{R}', k);$
- A3.  $\chi(\mathcal{R}, k) \geq \chi(\mathcal{R}, \ell);$
- A4. If  $\mathcal{R}$  and  $\mathcal{R}'$  differ only in that the items at positions  $k$  and  $\ell$  are exchanged, then  
 $\alpha(\mathcal{R}(k)) \leq \alpha(\mathcal{R}(\ell)) \iff \chi(\mathcal{R}, \ell) \geq \chi(\mathcal{R}', \ell);$
- A5.  $\chi(\mathcal{R}, k) \geq \chi(\mathcal{R}^*, k).$

## BubbleRank

**Methodology:** start with an *initial base list*  $\mathcal{R}_0$  and improve it online by gradually exchanging higher-ranked less attractive items for lower-ranked more attractive items.

- ▶ At each step  $t$ :
  - ▷  $h \leftarrow t \bmod 2$
  - ▷ For  $k \in [(K-h)/2]$ : // *Building the display list*  
 Randomly exchange items  $\mathcal{R}_t(2k-1+h)$  and  $\mathcal{R}_t(2k+h)$  in list  $\mathcal{R}_t$ , if  $s_{t-1}(i, j) \leq 2\sqrt{n_{t-1}(i, j) \log(1/\delta)}$
  - ▷ Display  $\mathcal{R}$  and observe clicks  $c_t \in \{0, 1\}^K$
  - ▷ For  $k \in [(K-h)/2]$  and  $i \leftarrow \mathcal{R}_t(2k-1+h)$ ,  $j \leftarrow \mathcal{R}_t(2k+h)$ : // *Update stats*  
 update  $s_t(i, j)$  and  $\mathbf{n}(i, j)$  if  $|c_t(2k-1+h) - c_t(2k+h)| = 1$
  - ▷ For  $k \in [(K-h)/2]$  and  $i \leftarrow \mathcal{R}_t(2k-1+h)$ ,  $j \leftarrow \mathcal{R}_t(2k+h)$ : // *Updating the base list*  
 Permanently exchange  $\mathcal{R}_{t+1}(k)$  and  $\mathcal{R}_{t+1}(k+1)$  if  $s_t(j, i) > 2\sqrt{n_t(j, i) \log(1/\delta)}$

## BubbleRank illustration

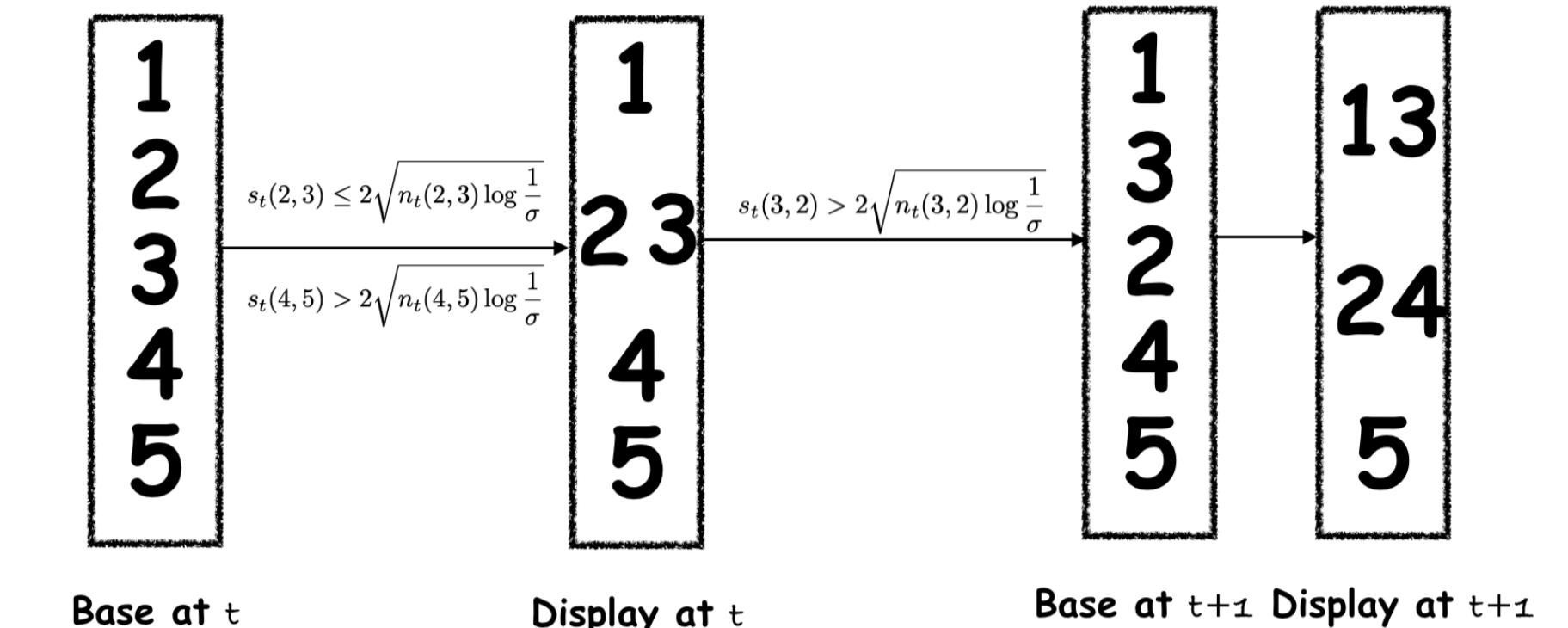


Figure: Illustration of BubbleRank with 5 items at step  $t$ .

## Main Results

**Theorem 1** (Upper Bound). The expected  $n$ -step regret of BubbleRank is bounded as

$$R(n) \leq 180K \frac{\chi_{\max} K - 1 + 2|\mathcal{V}_0|}{\chi_{\min}} \log(1/\delta) + \delta^{\frac{1}{2}} K^3 n^2.$$

**Lemma 2** (Safety). Let

$$\mathcal{V}(\mathcal{R}) = \{(i, j) \in [K]^2 : i < j, \mathcal{R}^{-1}(i) > \mathcal{R}^{-1}(j)\}$$

be the set of *incorrectly-ordered item pairs* in list  $\mathcal{R}$ . Then

$$|\mathcal{V}(\mathcal{R}_t)| \leq |\mathcal{V}(\mathcal{R}_0)| + K/2$$

holds uniformly over time with probability of at least  $1 - \delta^{\frac{1}{2}} K^2 n$ .

## Experimental setup

- ▶ Yandex Click log: at least one query in each session with 10 ranked items and 30M search sessions in total
- ▶ We randomly choose 100 frequent search queries and learn their CMs, DCMs and PBMs
- ▶  $L = 10$  items with  $K = 10$  positions
- ▶ Goal: place 5 most attractive items in the descending order of attractiveness at the 5 highest positions

## Experimental results

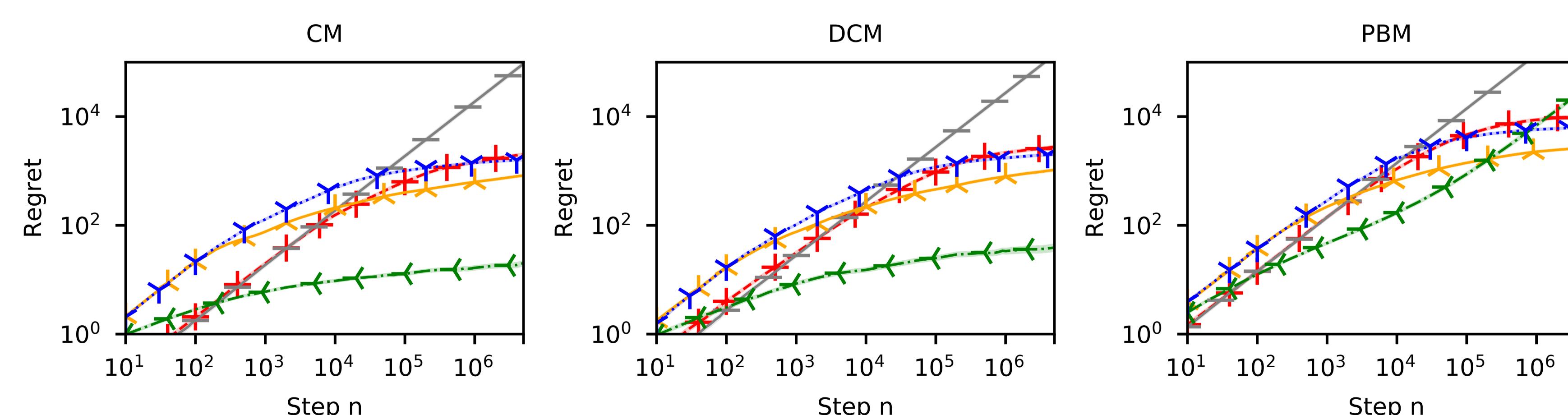


Figure: The  $n$ -step regret of BubbleRank (red), CascadeKL-UCB (green), BatchRank (blue), TopRank (orange), and Baseline (grey).

## Experimental results cont'd

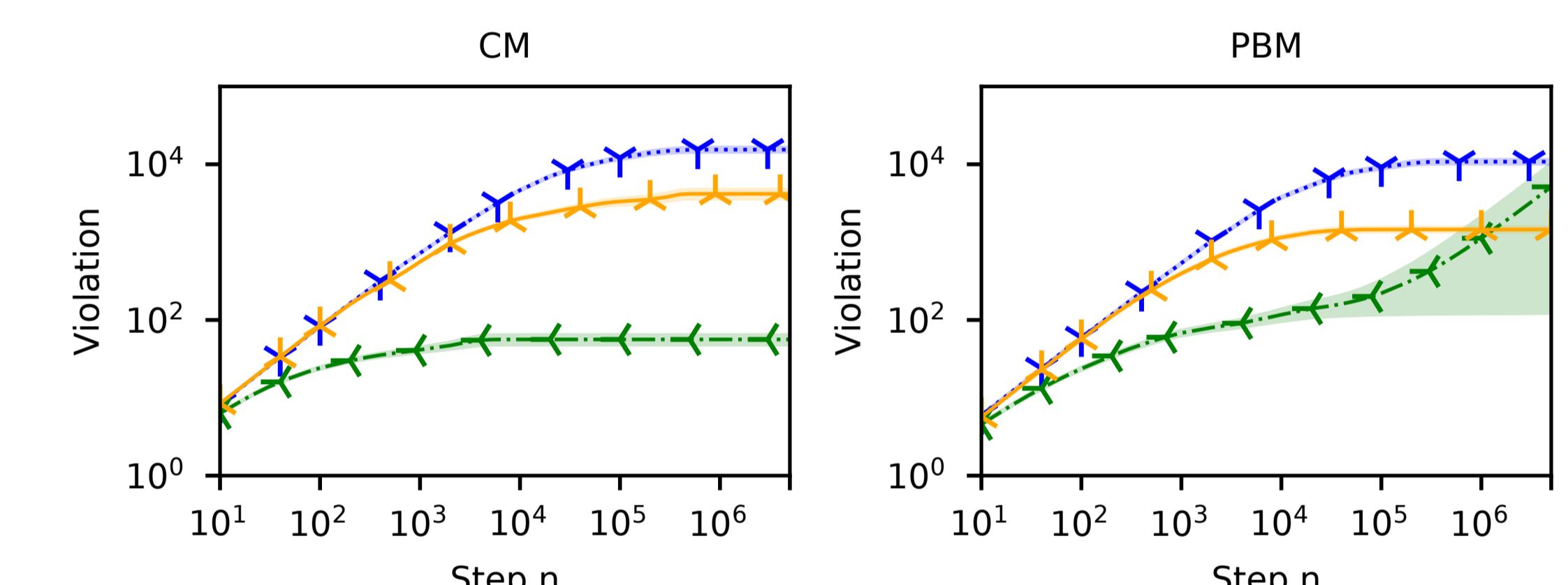


Figure: The  $n$ -step violation of the safety constraint.

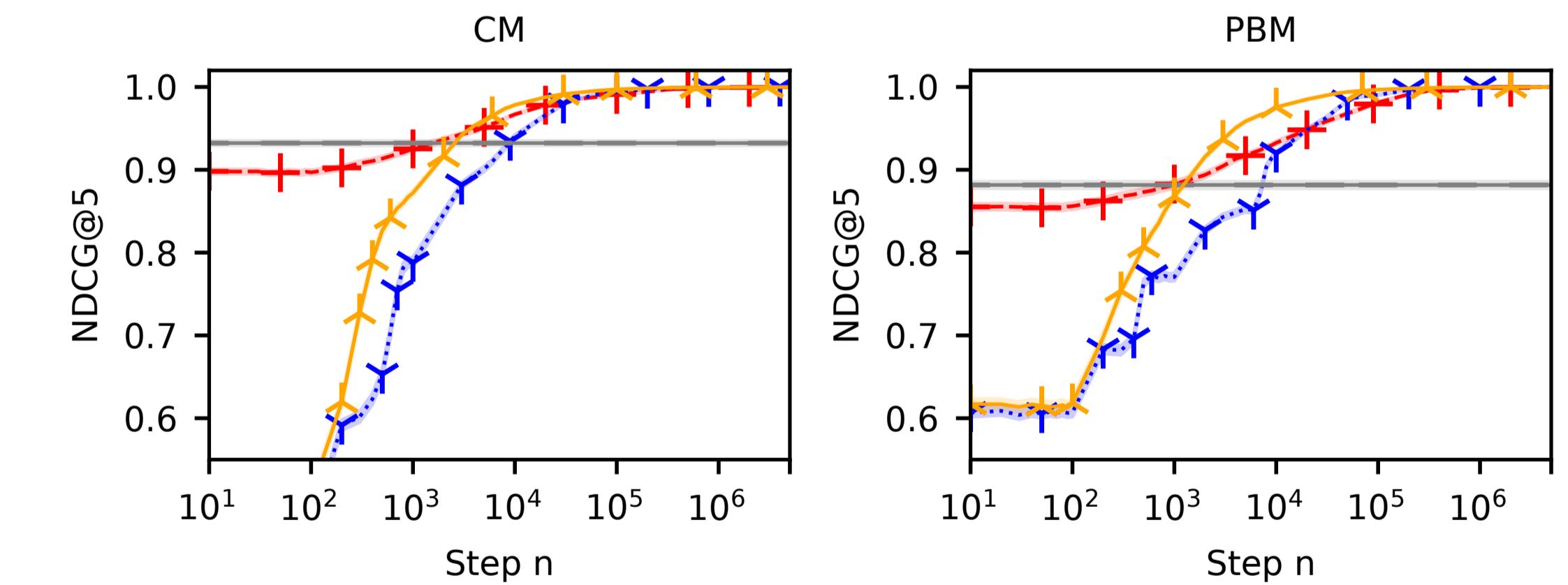


Figure: The per-step NDCG@5.

## Conclusion

- ▶ BubbleRank fills the gap between online and offline LTR approaches in literature
- ▶ BubbleRank explores under a safety constraint
- ▶ BubbleRank learns slower than TopRank but can learn the optimal ranking eventually
- ▶ Future work: further theoretical and experimental analysis on BubbleRank in the online learning to rank setup